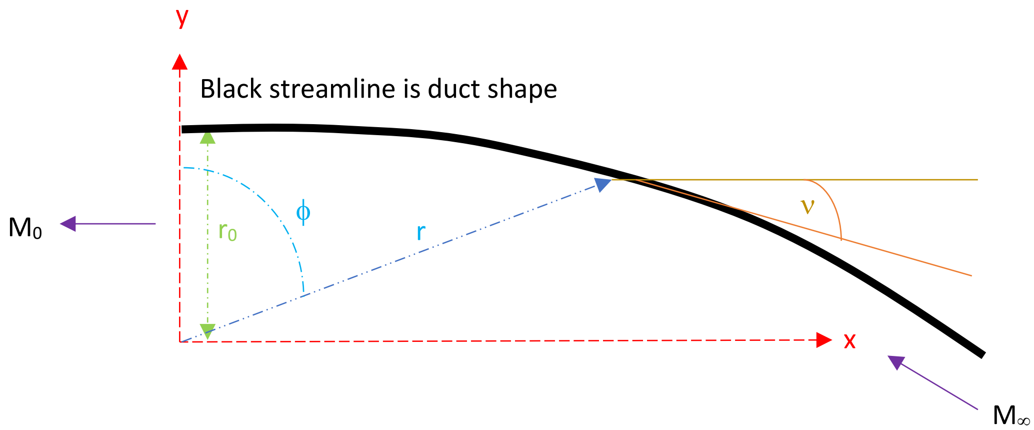
Design of external linear isentropic compression surfaces



This method works by using a Prandtl-Meyer expansion fan solution in reverse.

The correct shape for the duct is same as the flow expansion streamline around a sharp corner, the angle of which gives the correct input and output Mach numbers.

The method generates both polar (ϕ, r) and Cartesian (x, y) coordinates for the duct shape.

The variable inputs to the method are the duct height r_0 and the ratio of specific heats γ.

The algorithm runs through all the Mach numbers starting at exit M_0 and ending at the free stream M_∞, generating the coordinates at each one – the more coordinates you generate the more detail you have to draw your duct shape.

Here is the procedure (angles are in degrees):

1. Set r_0 (design decision) and γ (fluid conditions – usually 1.4)

2. Calculate the following constant for use in the equations:

$$K = \sqrt{\frac{(\gamma - 1)}{(\gamma + 1)}}$$

   For γ = 1.4, this is 0.406

3. You need to do the next steps for each Mach number M, between M_0 and M_∞ (the more steps you use the more points you'll have to draw the shape):

4. Calculate ϕ:

$$\phi = \frac{1}{K} tan^{-1} K\sqrt{M^2 - 1}$$

5. Calculate r:

$$r = \left(\frac{1}{(\cos(K\phi))^6}\right) r_0$$

6. You now have the polar coordinates (r, ϕ)

7. For each Mach number M, calculate the Mach angle μ:

$$\mu = sin^{-1}\frac{1}{M}$$

8. You can now calculate the flow angle ν at that point:

$$\nu = \phi + \mu - 90^o$$

9. Finally calculate x and y:

$$x = rcos(\mu - \nu)$$

$$y = rsin(\mu - \nu)$$

10. You now have the (x, y) coordinates.

Here is an example for you to check your calculations:

INPUTS	
Mach No =	1.3
Gamma =	1.4
r_0 =	1

OUTPUTS		
k =	0.40824829	
ϕ =	45.88546188	Polar coords
r =	1.386195875	Polar coords
μ =	50.28490524	
ν =	6.170367117	
x =	0.995219533	Rect coords
y =	0.964923357	Rect coords

References:

J. Seddon, E. L. Goldsmith, Intake Aerodynamics, AIAA. 1999 (2nd ed) – Note there is a missing equation in this reference, and it uses the next reference as a source (I don't think the authors have actually done the calculation).

J. F. Connors, R. G. Meyer, Design Criteria for Axisymmetric and Two-Dimensional Supersonic Inlets and Exits, NACA, Technical note 3589, 1956. (this is available on-line).